The Mathemagician's

Handbook

Maths Language

In maths there are many ways of saying the same thing. It is important to learn these all.

+	**-**
add plus altogether increase find the total *calculate the perimeter*	subtract minus how many less find the difference decrease take away deduct
X	**÷**
multiply by times lots of product *calculate the area* *calculate the volume*	divide share split into equal groups how many times goes into

❖ **DON'T FORGET**

+and**–** are simply the opposite of each other.

If you know $24 + \square = 50$

then it follows that 50 - 24 will give the missing answer.

In the same way, **x** and **÷** are also opposites.

If you know $\square \times 6 = 138$

then it could be easier to think $\begin{array}{r} 2\;3 \\ 6\overline{)1\;3^{1}8} \end{array}$ to calculate the answer.

Square Numbers and Cube Numbers

The following table shows all the square and cube numbers you should know quickly.

	Square Numbers		Cube Numbers	
1	1 x 1	1	1 x 1 x 1	1
2	2 x 2	4	2 x 2 x 2	8
3	3 x 3	9	3 x 3 x 3	27
4	4 x 4	16	4 x 4 x 4	64
5	5 x 5	25	5 x 5 x 5	125
6	6 x 6	36		
7	7 x 7	49		
8	8 x 8	64		
9	9 x 9	81		
10	10 x 10	100	10 x 10 x 10	1000
11	11 x 11	121		
12	12 x 12	144		

- Apart from the number 1, which is the only square number that is also a cube number?

Multiples and Factors

- A **multiple** is a number multiplied. Some multiples of 10 are 20, 30, 40, 50 because you multiply 10 by another number to make the larger number.

- A **factor** is a number that will divide equally into a bigger number. 2 and 5 are factors of 10.

Prime Numbers

A prime number can be divided evenly only by 1 or itself and it must be a whole number greater than 1.

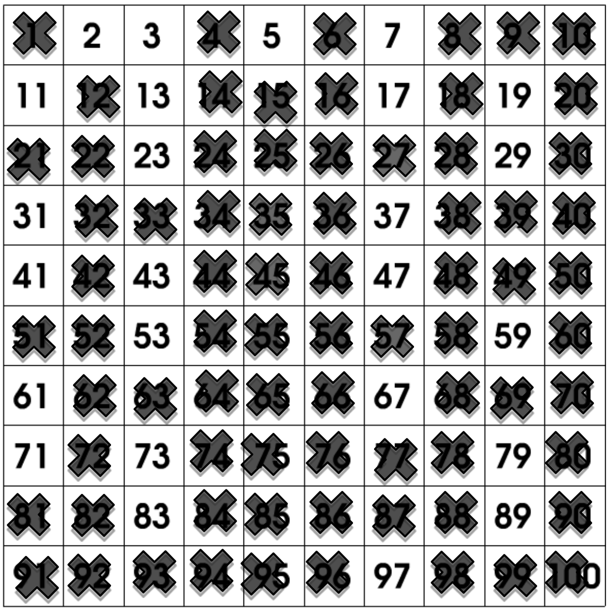
Remember the rule: It’s easy to check if a number under 100 is a prime number. You only have to work out if it divides evenly by 2, 3, 5 or 7.

Do these 3 steps:

- Step 1 - all x2 are even numbers – (0,2,4,6,8 units)
- Step 2- all x5 numbers end in 0 or 5
- Step 3- check if the number divides evenly by 3 or 7. If not, then it’s a prime number.

Find all the Prime Numbers less than 100.

- Cross out 1
- Cross out all numbers that ÷2, ÷3, ÷5, ÷7
- All numbers left are prime numbers



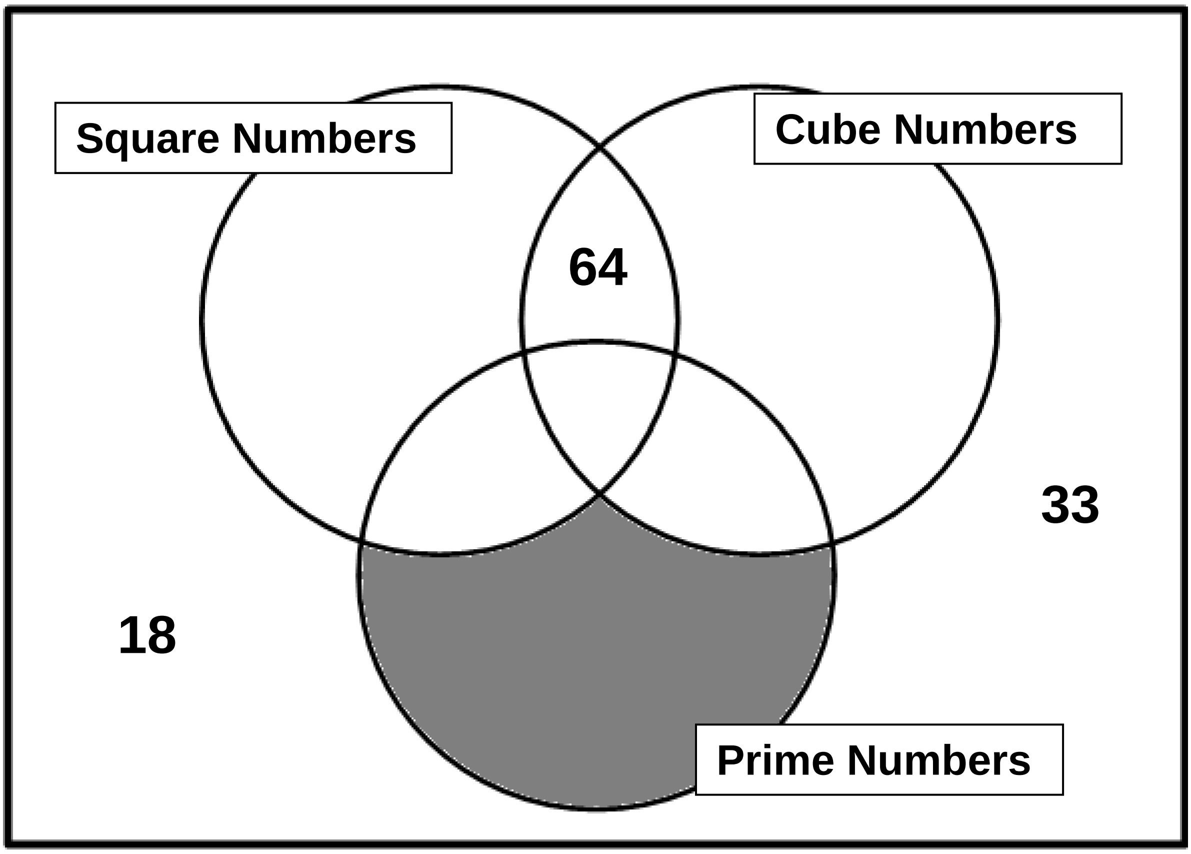
~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Now see if these are prime numbers.

101 237 256 462

293 735 1028 771

Squares, Cubes and Primes in Venn Diagrams



- Don't forget…..
    - It is impossible for a prime number to go in any section other than the one shaded in grey above.
    - 64 is the ONLY number that will ever go in the area shown above.
    - If a number does not fit in any of the circles then it should be written outside the circles but inside the box. Look at the examples above -18 and 33.

Averages

- To find the average of a set of numbers, simply add up the numbers and divide by the amount of numbers you have added.

e.g. Find the average of these 5 numbers: 38 27 51 16 43

38 + 27 + 51 + 16 + 43 = 175
Now divide 175 by 5 (as you had 5 numbers at the start.)
The average is 35

- To find a missing number from a list of numbers when you know the average you need to first work out the total then subtract the numbers you already know.

e.g 5 children measure their height then calculate their average height. Their average height is 143cm.

Tim is 142cm tall Peter is 153cm tall
Holly is 137cm tall Jane is 144cm tall
How tall is Andrew?

To answer this question you need to find out how tall the 5 children are altogether, then subtract the heights you already know.

143cm x 5 = 715cm.

Now subtract the heights of the children that you already know (142cm, 153cm, 137cm and 144cm) and you will be left with Andrew's height – 139cm.

Connecting Numbers

Weight Facts	Length Facts
There are 1000 grams in 1 kilogram.	There are 1000 metres in 1 kilometre.
1g is the same as $^{1}/_{1000}$of 1 kg.	1m is the same as $^{1}/_{1000}$of 1 km.
10g is the same as $^{1}/_{100}$ of 1 kg.	10m is the same as $^{1}/_{100}$ of 1 km.
100g is the same as $^{1}/_{10}$of 1 kg.	100m is the same as $^{1}/_{10}$of 1 km.
250g is the same as $^{1}/_{4}$ of 1 kg.	250m is the same as $^{1}/_{4}$ of 1 km.
500g is the same as $^{1}/_{2}$ of 1 kg.	500m is the same as $^{1}/_{2}$ of 1 km.
750g is the same as $^{3}/_{4}$ of 1 kg.	750m is the same as $^{3}/_{4}$ of 1 km.
To change g into kg you need to divide the number of g by 1000. e.g. 2000g = 2kg 4500g = 4.5kg	To change m into km you need to divide the number of m by 1000. e.g. 2000m = 2km 4500m = 4.5km

Notice the similarities between weight and length measurements.

Did you know…..?
The prefix **kilo-** means 1000. When you have 1000 smaller measures it's the same as 1 "**kilo-**" measure.

The prefix **milli-** means 1000. It is different to **kilo**- because it really means 1 thing split into 1000 smaller pieces.

Fractions	Decimals	Percentages	Fraction of a metre	Centimetres
¼	0.25	25%	¼ of a metre	25 cm
½	0.5	50%	½ of a metre	50 cm
¾	0.75	75%	¾ of a metre	75 cm
$^{1}/_{5}$	0.2	20%	$^{1}/_{5}$of a metre	20 cm
$^{1}/_{10}$	0.1	10%	$^{1}/_{10}$of a metre	10 cm

There are 100 centimetres in a metre.
There are 10 millimetres in a centimetre.
There are 1000 millimetres in a metre.

Did you know…..?

The prefix **cent-** means 100.

LINES

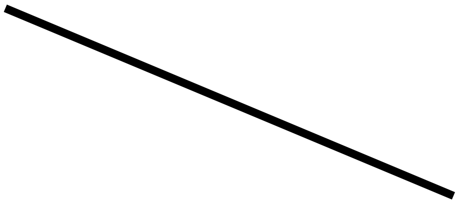
HORIZONTAL

A line 'straight across' (parallel to the Earth's horizon)

VERTICAL

A line straight
'up and down'
(at right angles to
the Earth's horizon)

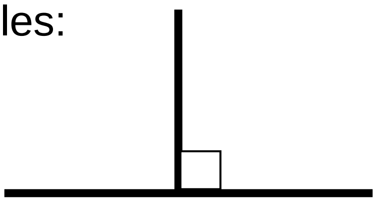
A line joining opposite
corners in a shape

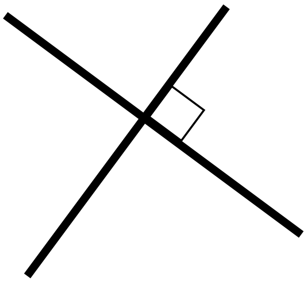


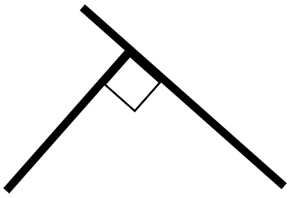
OBLIQUE a sloping or slanted line

PERPENDICULAR lines that meet or cross at right angles to each other.

Examples:

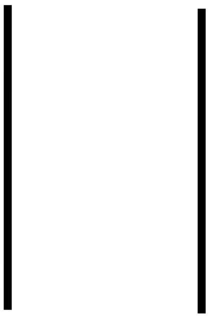


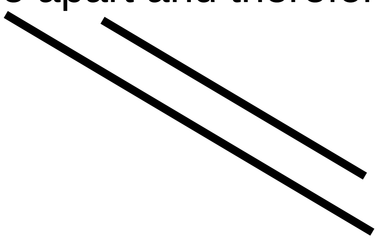




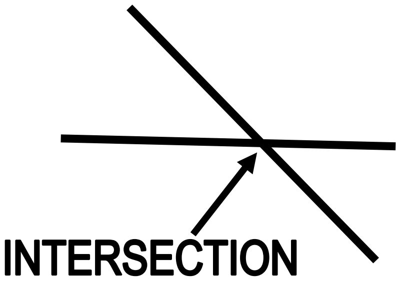
PARALLEL lines always remain the same distance apart and therefore never meet.

Examples:





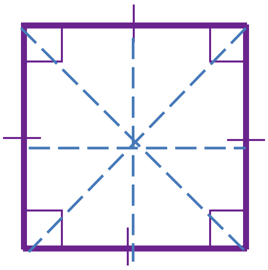
The point where lines meet or cross is called the **INTERSECTION.**



Quadrilaterals

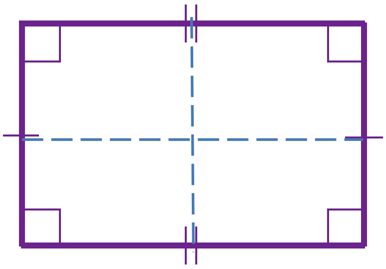
A QUADRILATERAL is a flat shape with FOUR sides. The angles inside all quadrilaterals add up to 360 °.

SQUARE



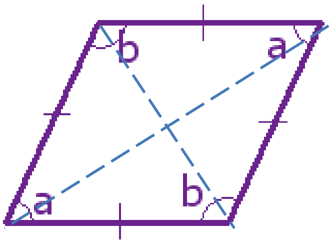
- all four sides are equal in length
- all four angles are right angles
- opposite sides are parallel
- 4 lines of symmetry

RECTANGLE



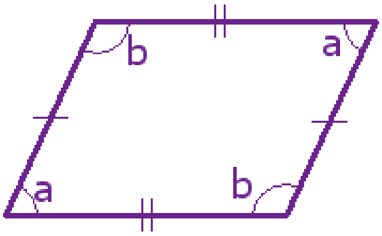
- opposite sides are equal inlength
- all four angles are right angles
- opposite sides are parallel
- 2 lines of symmetry

RHOMBUS



- all four sides are equal in length
- 2 acute and 2 obtuse angles
- opposite angles are equal
- opposite sides are parallel
- 2 lines of symmetry

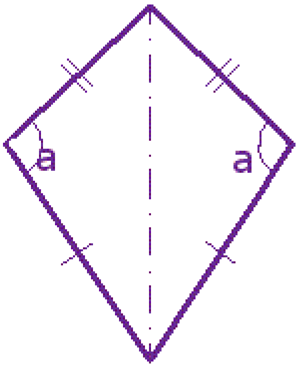
PARALLELOGRAM



- opposite sides are equal in length
- 2 acute and 2 obtuse angles
- opposite angles are equal
- opposite sides are parallel
- NO lines of symmetry

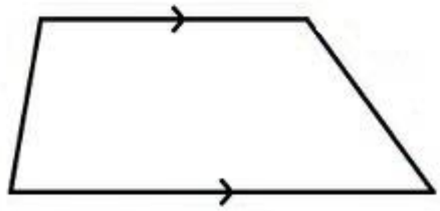
More Quadrilaterals

KITE



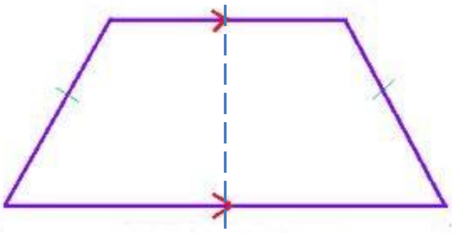
- 2 pairs of adjacent sides that are equal in length
- one pair of equal opposite angles
- no sides are parallel
- 1 line of symmetry

TRAPEZIUM



- no sides are equal in length
- no equal angles
- one pair of parallel sides
- no lines of symmetry

ISOSCELES TRAPEZIUM



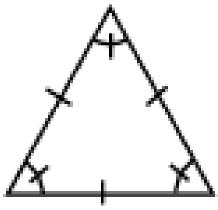
- one pair of sides are equal in length
- two pairs of adjacent angles are equal
- one pair of parallel sides
- one line of symmetry

- **NB** Adjacent angles are those that are next to each other.

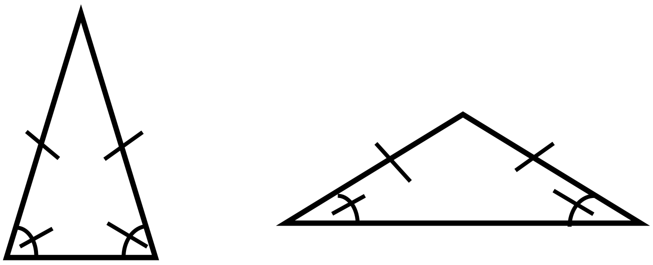
TRIANGLES

A TRIANGLE is a flat shape with THREE sides.The angles inside all triangles add up to 180°.

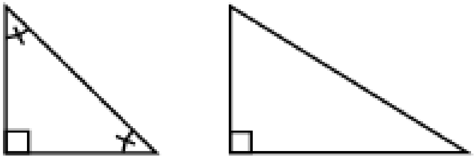
These are the 4 different types of triangles.



EQUILATERAL
- all three sides are equal
- all angles are 60°
- 3 lines of symmetry

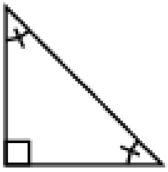


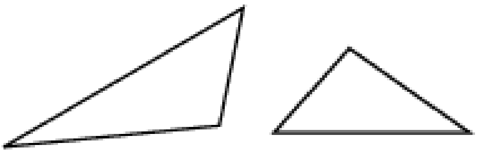
ISOSCELES
- two sides equal in length
- two equal angles
- one line of symmetry



RIGHT-ANGLED
- contains one right angle

This right-angled triangle is also isosceles because it has 2 sides the same length and 2 equal angles.





SCALENE
- all three sides are different lengths
- NO equal angles
- NO lines of symmetry

A **POLYGON** is a flat shape with three or more straight sides. The following is a list of names of polygons and the number of straight sides they have.

PENTAGON ~ 5 sides
HEXAGON ~ 6 sides
OCTAGON ~ 8 sides

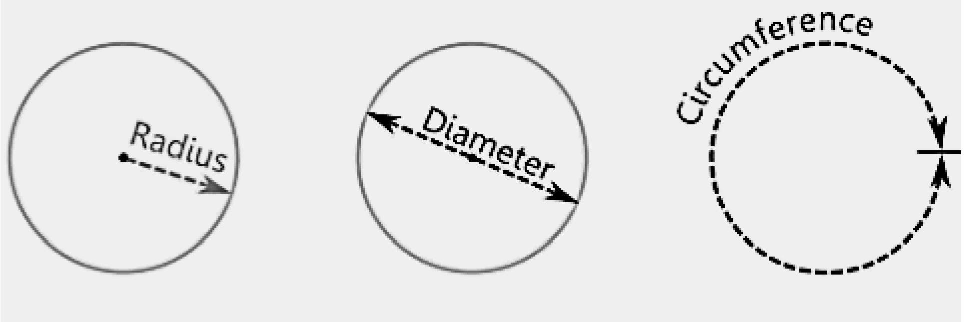
MOST COMMON

HEPTAGON ~ 7 sides
NONAGON ~ 9 sides
DECAGON ~ 10 sides

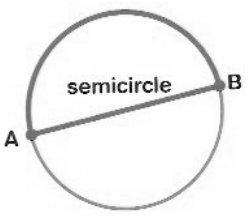
LESS COMMON

A **REGULAR** shape has <u>all its sides equal in length</u> and all its angles are equal. A regular shape will have the same number of lines of symmetry as it does sides.

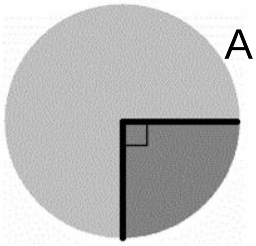
The Circle



- A **RADIUS** is a line from the centre of a circle to its outside edge.
- A **DIAMETER** is a line which runs through the centre of a circle and divides it into two **SEMI-CIRCLES.** The DIAMETER is always twice as long as the RADIUS.
- The **CIRCUMFERENCE** is the outside edge of a circle



A **semicircle** is half a circle.

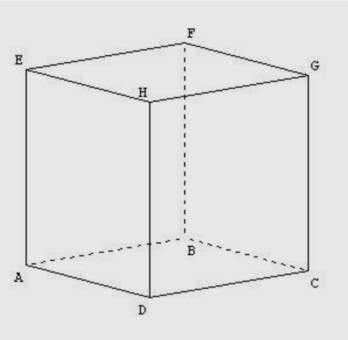


A **quadrant** is a quarter of a circle

SOLID SHAPES

Solid shapes are also called 3-Dimensional or 3-D shapes because they have 3 dimensions - length, width and height.

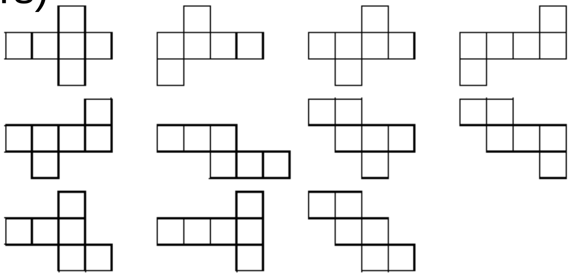
The following are 3D shapes and their properties



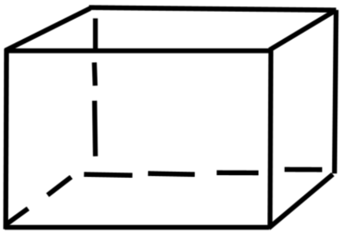
CUBE

- 6 square faces
- 8 vertices (corners)
- 12 edges

Nets of cubes:

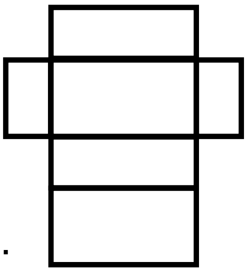


CUBOID

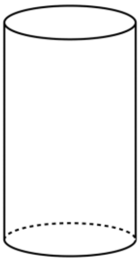


- 6 faces (6 rectangles or 4 rectangles and 2 squares)
- 8 vertices (corners)
- 12 edges

Example of a cuboid net:

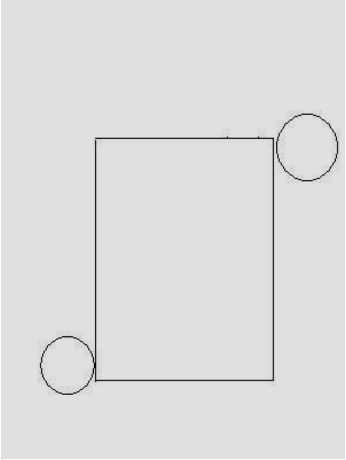


CYLINDER

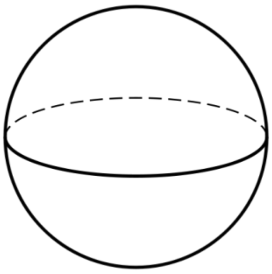


- 2 flat faces (circular)
- 1 curved surface
- 2 curved edges, no vertices
- will roll

a cylinder net:

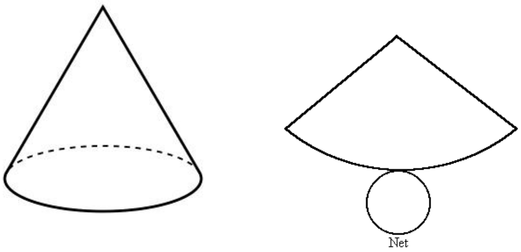


SPHERE

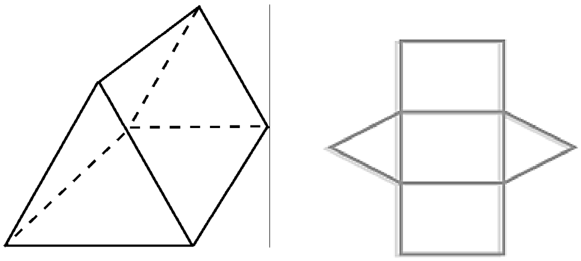


- a ‘ball’ shape
- one perfectly curved surface
- no vertices or straight edges
- will roll

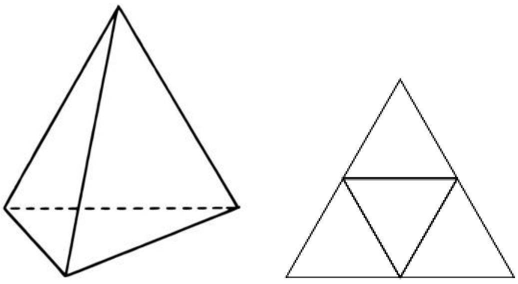
More 3-D Shapes



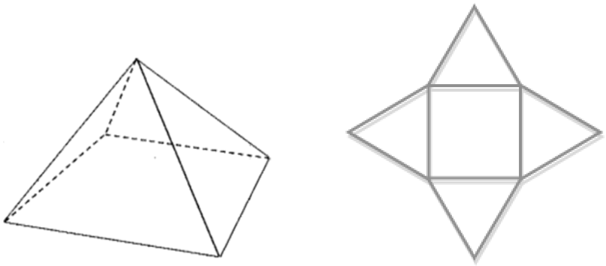
CONE
- 1 flat circular face
- 1 curved surface
- 1 curved edge
- 1 vertex



TRIANGULAR PRISM
- 5 faces (3 rectangles and 2 triangles)
- 6 vertices
- 9 straight edges



TRIANGULAR BASED PYRAMID
or TETRAHEDRON
- 4 faces
- 4 vertices
- 6 straight edges



SQUARE BASED PYRAMID
- 5 faces (4 triangles and 1 square)
- 5 vertices
- 8 straight edges

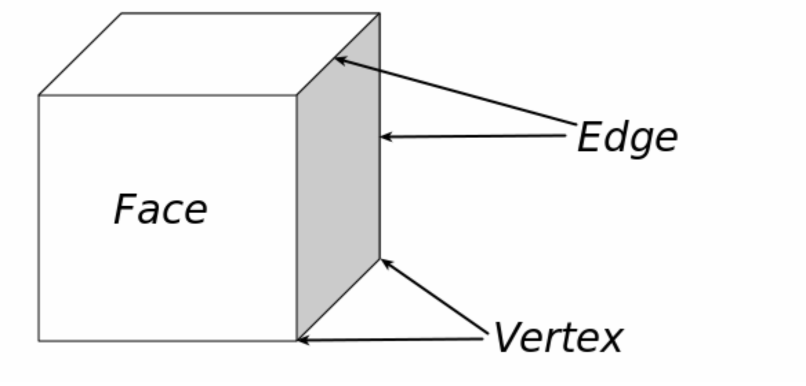
All these solid shapes belong to either the prism or pyramid family.
A **PRISM** keeps its shape all along its length.
A **PYRAMID** narrows to reach a point at the top.

3-D Shapes – Faces, Edges & Vertices

The faces are the flat surfaces of the shape.

The edge of a 3-D shape is the name given where 2 sides meet in the shape.

The vertex (vertices is the plural) is where the corners of the shape meet.



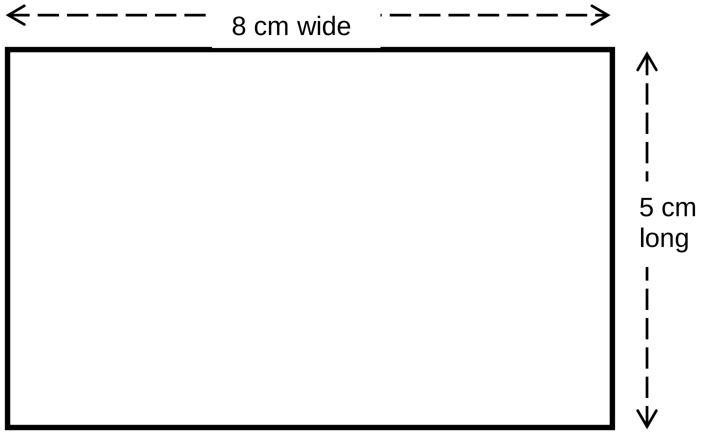
These are the properties of shapes you need to know.

Shape	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Triangular prism	5	9	6
Triangular based pyramid	4	6	4
Square based pyramid	5	8	5
Cylinder	3	2	0
Cone	2	1	1

AREA

AREA means the amount of space a **flat** shape takes up – like the **surface** of something e.g. your desk or the seat of your chair.

To work out the area of a shape like this, you measure its length and width (breadth) then multiply together these two measurements.
For example: This rectangle measures 8cm wide and 5 cm long.



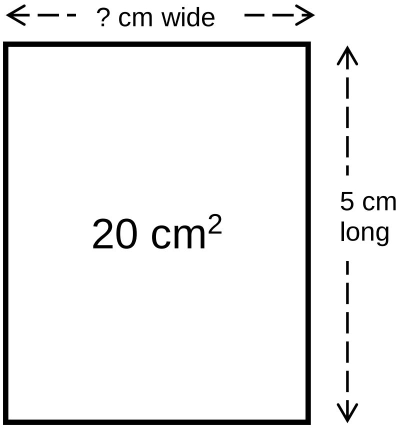
length = 5cm

width = 8 cm

Area: 5 x 8 = 40cm^2

- Don't forget you must ALWAYS write your answer as the measurement squared.

e.g. cm^2
m^2 this is the symbol representing "squared"
km^2



- Sometimes you are asked to calculate the length of a side given the area and the length of the other side.

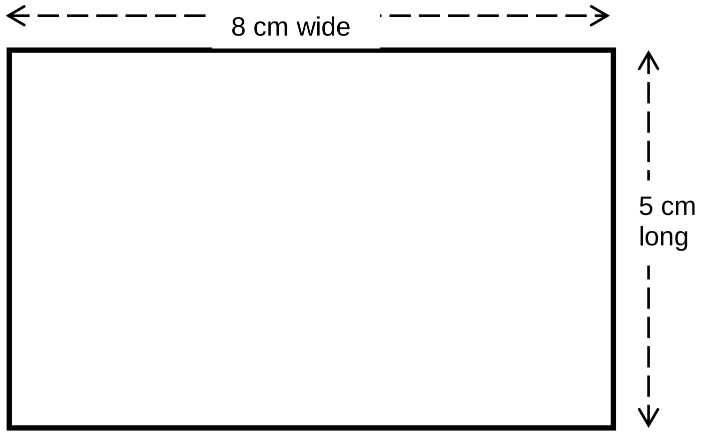
To do this, simply reverse the calculation.

5cm x ☐ cm = 20cm^2

20cm^2÷ 5cm = 4cm. – The other side is 4cm.

PERIMETER

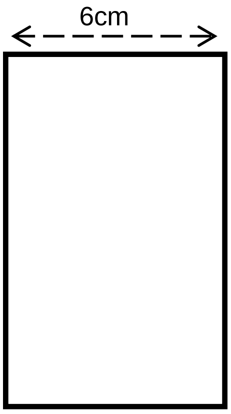
PERIMETER means the distance around a space – like the length of a fence around a field.



This rectangle has 2 sides that are 8cm long and 2 sides that are 5cm long.

That means its perimeter is 8cm + 8cm + 5cm+ 5cm = 26cm

Sometimes you are given the perimeter and the length of one side and you are asked to calculate the area of a rectangle.



e.g. a rectangle has a perimeter of 32cm. One side is 6cm long. Calculate the area of the rectangle.

Start by calculating the length of the other sides.

- Opposite sides are the same length, so double the length you already know and subtract it from the length of the perimeter.
    - 6cm x2 = 12cm
    - 32cm - 12cm = 20 cm

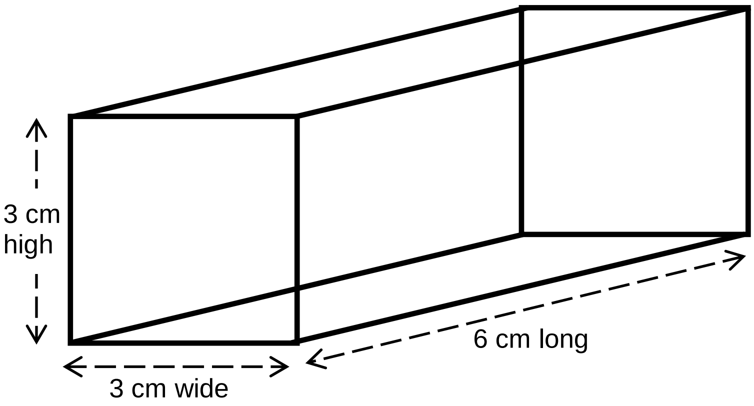
- Now divide that number in half and you will have the length of both long and short sides of the rectangle.
    - 20cm ÷ 2 = 10cm
    - The sides of the rectangle are 6cm and 10 cm
    - Multiply the 2 lengths together to find the area.
    - 6cm x 10cm = 60cm^2
- If you know the perimeter of a shape and need to find the area, simply work backwards.

VOLUME

<u>VOLUME</u> means the amount of space that is taken up by a container.

To work out the volume of a container, you measure its length, width (breadth) and height, then multiply together these three measurements.

<u>For example:</u> This cuboid measures 3cm high, 3cm wide and 6 cm long.



height = 3 cm

width= 3 cm

length= 6 cm

Volume:

3 x 3 x 6 = 54cm^3

- <u>Don't forget</u> you must ALWAYS write your answer as the measurement cubed.

e.g. cm^3
m^3 this is the symbol representing "cubed"
km^3

Fractions, Decimals and Percentages

- The word percent simply means ‘out of 100’
- A percentage is just like a fraction.
- This is the symbol for percent - %
- We can write 1% like this or as a fraction like this $^{1}/_{100}$
- A decimal is another way of writing a fraction or a percentage.
- Decimals and fractions are always worth less than 1.

Percent	Decimal	Fraction	Lowest Terms
1%	0.01	$^{1}/_{100}$	
5%	0.05	$^{5}/_{100}$	$^{1}/_{20}$
10%	0.1	$^{10}/_{100}$	$^{1}/_{10}$
12½%	0.125	$^{12½}/_{100}$	$^{1}/_{8}$
20%	0.2	$^{20}/_{100}$	$^{1}/_{5}$
25%	0.25	$^{25}/_{100}$	$^{1}/_{4}$
30%	0.3	$^{30}/_{100}$	$^{3}/_{10}$
$33^{1}/_{3}\%$	0.333...	$^{331/3}/_{100}$	$^{1}/_{3}$
40%	0.4	$^{40}/_{100}$	$^{2}/_{5}$
50%	0.5	$^{50}/_{100}$	$^{1}/_{2}$
60%	0.6	$^{60}/_{100}$	$^{3}/_{5}$
70%	0.7	$^{70}/_{100}$	$^{7}/_{10}$
75%	0.75	$^{75}/_{100}$	$^{3}/_{4}$
80%	0.8	$^{80}/_{100}$	$^{4}/_{5}$
90%	0.9	$^{90}/_{100}$	$^{9}/_{10}$
99%	0.99	$^{99}/_{100}$	$^{99}/_{100}$
100%	**1**	$^{100}/_{100}$	

- Always remember to simplify fractions to the lowest possible terms.

How to simplify fractions

There are two ways to simplify a fraction:

Method 1

Try dividing both the top and bottom of the fraction until you can't go any further (try dividing by 2,3,5,7,...etc).

Example: Simplify the fraction $^{24}/_{108}$:

$$\frac{24}{108} \underset{\div 2}{\overset{\div 2}{=}} \frac{12}{54} \underset{\div 2}{\overset{\div 2}{=}} \frac{6}{27} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{9}$$

Method 2

Divide both the top and bottom of the fraction by the Greatest Common Factor, (you have to work it out first!).

Example: Simplify the fraction $^{8}/_{12}$:

1. The largest number that goes exactly into both 8 and 12 is 4, so *the Greatest Common Factor is 4*.

2. Divide both top and bottom by 4:

$$\frac{8}{12} \underset{\div 4}{\overset{\div 4}{=}} \frac{2}{3}$$

And the answer is: $^{2}/_{3}$

Simplifying Fractions

Points to remember

- If both numbers in the fraction end with a ‘0’ then 10 will divide into both of them e.g. $^{10}/_{100} \longrightarrow {}^{1}/_{10}$
- If both numbers end with a‘5’ then 5 will divide into them. e.g. $^{5}/_{25} \longrightarrow {}^{1}/_{5}$
- If both numbers end with a ‘0’ and a ‘5’ then 5 will divide into them. e.g. $^{15}/_{100} \longrightarrow {}^{3}/_{20}$
- If both numbers are even then 2 will divide into them. e.g. $^{16}/_{24} \longrightarrow {}^{8}/_{12} \longrightarrow {}^{4}/_{6} \longrightarrow {}^{2}/_{3}$
- Also remember your number facts from your times tables for more unusual fractions. e.g. $^{12}/_{30} \xrightarrow[\text{(both divide by 6)}]{} {}^{2}/_{15}$

Now practise bringing these fractions down to their lowest terms:

$^{75}/_{100}$	$^{18}/_{100}$	$^{25}/_{100}$	$^{10}/_{100}$	$^{62}/_{100}$
$^{50}/_{100}$	$^{20}/_{100}$	$^{85}/_{100}$	$^{40}/_{100}$	$^{45}/_{100}$

How to find the percent of a number

e.g. Find 75% of 256.

- Step 1 – look at the percentage
- Step 2 – change it to a fraction
- Step 3 – check the fraction is in its lowest terms and if not then simplify it.
- Step 4 – when the fraction is in its lowest terms, divide the number by the bottom part of your fraction.
- Step 5 – finally use the answer you have just got and multiply it by the top part of your fraction.

The answer you have just worked out is the percent of your starting number.

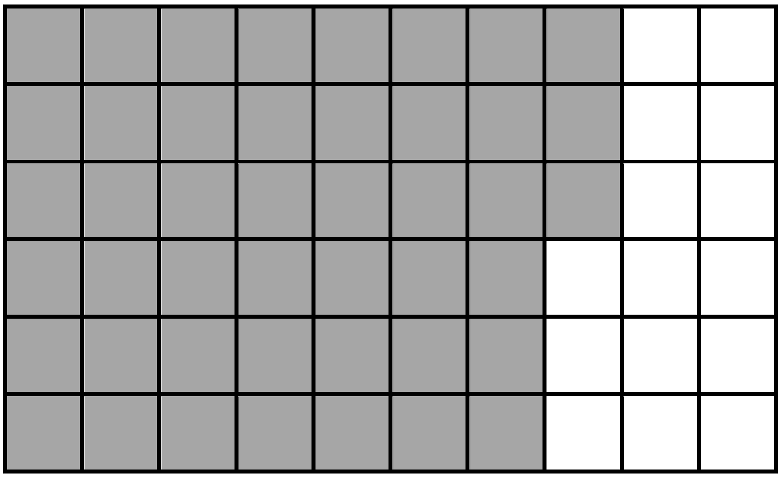
SO: to find 75% of 256

- Step 1 – 75%
- Step 2 – $^{75}/_{100}$
- Step 3 – $^{3}/_{4}$
- Step 4 – $\begin{array}{r} 6\ 4 \\ 4\overline{)2\ 5\ 6} \end{array}$
- Step 5 – $\begin{array}{r} 6\ 4 \\ \times\ 3 \\ \hline 1\ 9\ 2 \end{array}$

So now you have worked out that 75% of 256 is192

Percentages – Shading Squares

Example: Draw a shape made up of 60 squares and shade in 75%.



75% is ¾

- We find ¾ of 60
- 60 ÷ 4 = 15
- 15 x 3 = 45
- So 75% of 60 is 45
- We shade 45 squares

- How many squares are shaded? = 45
- How many squares are unshaded? = 15
- What % of the shape is shaded? = 75%
- What % of the shape is unshaded? = 25%

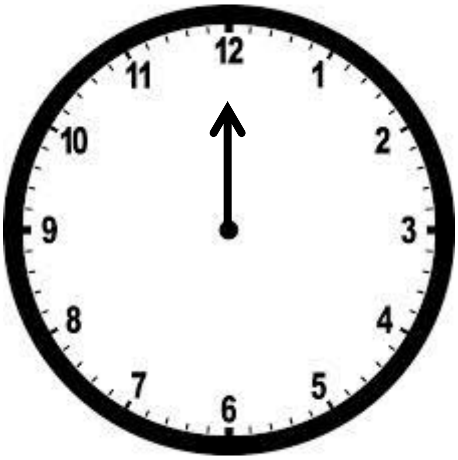
Now you try these.
Draw the shapes into your book, shade in the correct number of squares and answer the 4 questions about the shapes in the way I have done above.

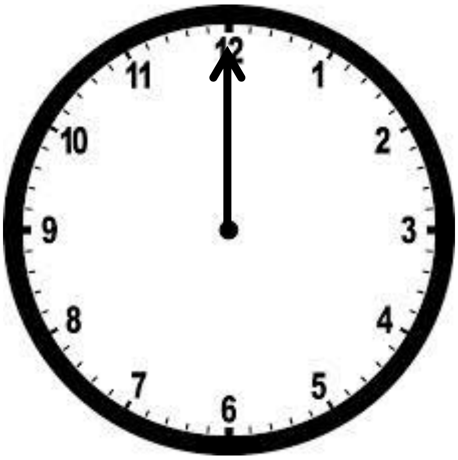
a) 45 squares and shade 20%
b) 40 squares and shade 75%
c) 50 squares and shade 50%
d) 48 squares and shade 25%
e) 30 squares and shade 10%
f) 55 squares and shade 40%
g) 60 squares and shade 90%
h) 70 squares and shade 70%

Time Facts

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
14 days = 1 fortnight
15 minutes = ¼ of an hour
30 minutes = ½ an hour
45 minutes = ¾ of an hour

52 weeks = 1 year
12 months = 1 year
365 days = 1 year
366 days = 1 leap year
(once every 4 years)
5 minutes = $^{1}/_{12}$ of an hour
5 minutes = $^{1}/_{12}$ of 360° = 30°
5 minutes = $^{1}/_{3}$ of a right angle





REMEMBER

On a clock, the short hand is the hour hand. Notice how you can easily see the number to which it is pointing.

The long hand is the minute hand and tells you how many minutes have passed since the time was at o'clock. Notice how the hand is touching the 12 in the picture.

24 Hour Clock

For one full day to pass, the hour hand (the small hand) on a clock must go around the clock face TWICE.

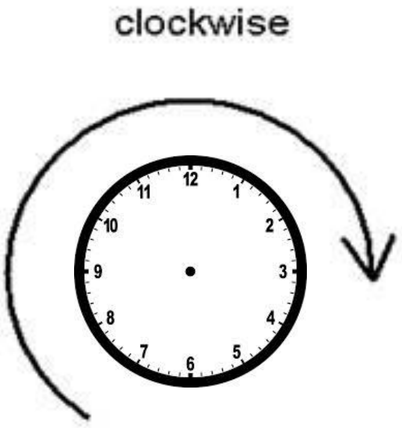
From midnight → to noon
and then from noon → to midnight
That's 2 sets of 12 hours which makes 24 hours = 1 day.

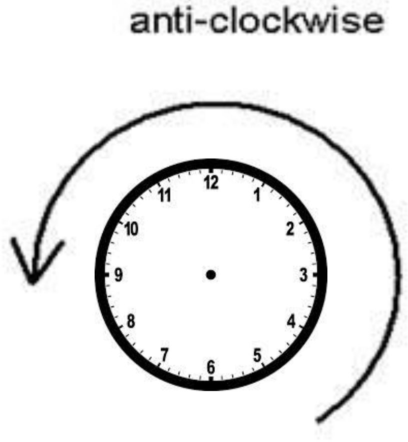
In 24 hour time the names of the times are not repeated – we just keep counting the hours that have passed from midnight until we return to 0.

MORNING	**AFTERNOON**
12:00 am (midnight) → 0000 hours	12:00 pm (noon) → 1200 hours
1:00 am → 0100 hours	1:00 pm → 1300 hours
2:00 am → 0200 hours	2:00 pm → 1400 hours
3:00 am → 0300 hours	3:00 pm → 1500 hours
4:00 am → 0400 hours	4:00 pm → 1600 hours
5:00 am → 0500 hours	5:00 pm → 1700 hours
6:00 am → 0600 hours	6:00 pm → 1800 hours
7:00 am → 0700 hours	7:00 pm → 1900 hours
8:00 am → 0800 hours	8:00 pm → 2000 hours
9:00 am → 0900 hours	9:00 pm → 2100 hours
10:00 am → 1000 hours	10:00 pm → 2200 hours
11:00 am → 1100 hours	11:00 pm → 2300 hours

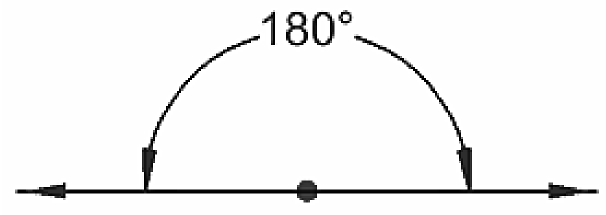
NB: It is very important that you don't forget to use the am or pm when using 12 hour clock to tell the difference between morning and afternoon.

Turning and Angles



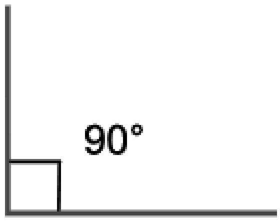


When we think about angles we are really talking about the amount of turning there is between two lines that are joined at a common point. Don't forget clockwise is the direction the hands on a clock move as times passes and anti-clockwise is the opposite.



The angle on a straight line is always 180° - look at the arrow heads in the picture. If you turned one of the lines from the central point, then the arrow head would have to turn through 180° to end up on top of the other one.

This is a right angle. It is a turn of 90°.

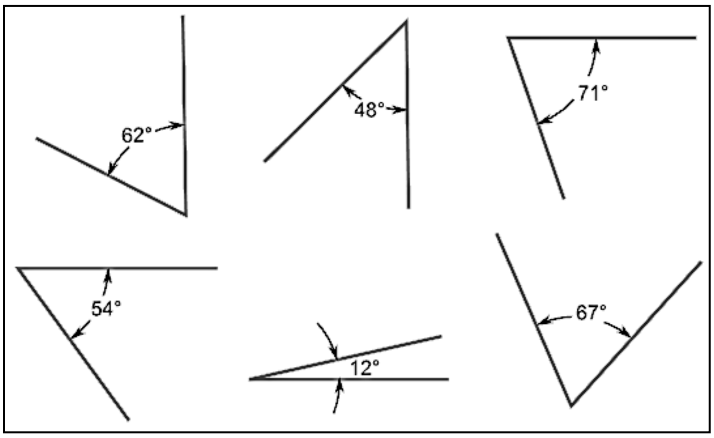


Useful facts to remember

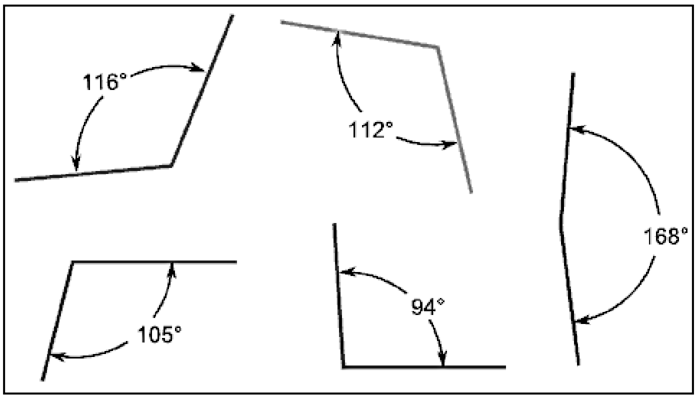
360° = 1 full turn or rotation. It is the same as 4 right angles.
270° = ¾ of a full turn or rotation. It is the same as 3 right angles.
180° = ½ a full turn or rotation. It is the same as 2 right angles.
180° is known as a straight angle.
90° = ¼ of a full turn or rotation. It is known as a right angle.

Other Types of Angles

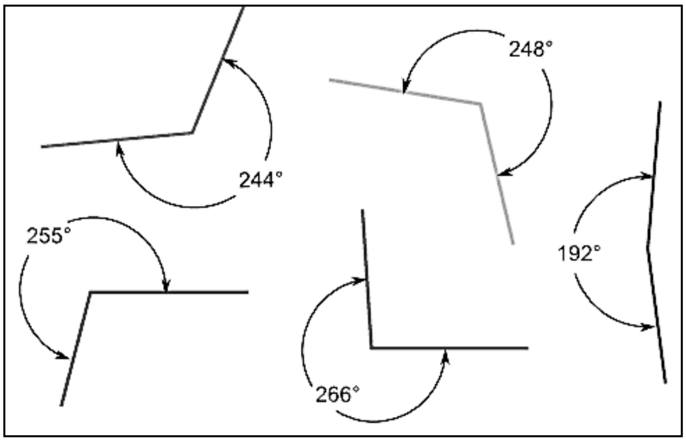
ACUTE angles are any angles that are smaller than a right angle. That means any angles less than 90°. Here are some examples of acute angles.



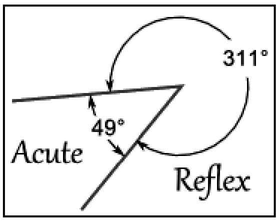
OBTUSE angles are any angles that are greater than a right angle, but smaller than a straight angle. That means any angles larger than90° but smaller than 180°. Here are some examples of obtuse angles.

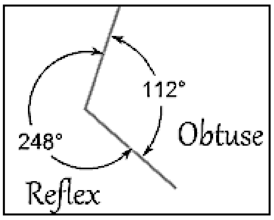


REFLEX angles are those that are greater than a straight angle. That means, more than 180°. Here are some reflex angles.



NB
All acute and obtuse angles have a reflex angle on their outside.





Angles– finding a missing angle

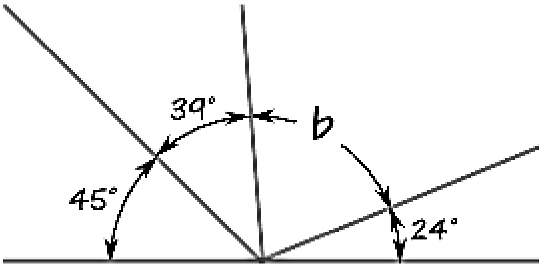
When you need to work out the size of a missing angle, you need to use the information you already know.

REMEMBER

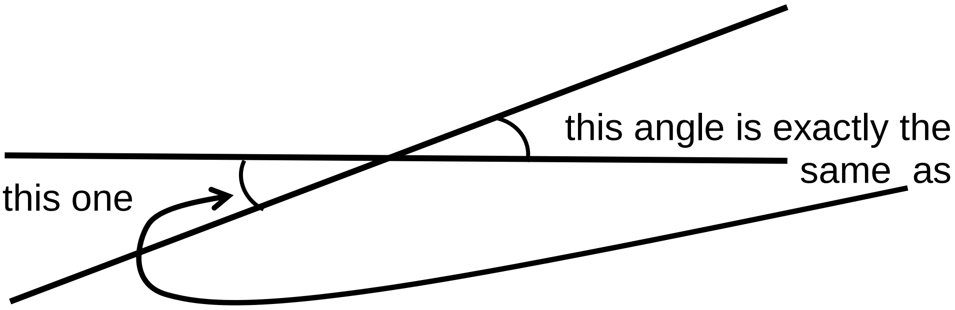
- All the angles inside a triangle add up to 180°
- All the angles inside a quadrilateral add up to 360°

- The angles on a straight line always add up to 180°. It doesn't matter how many angles there are!

  e.g. You find the size of angle b by subtracting the other angles from 180: 45° +39° +24° = 108°
  180° - 108° = 72° so b must be 72°



- Diagonally opposite angles are always the same.



Understanding Scale

Scale is when you draw a plan on paper of something which will be large in reality.

e.g. If you wanted to make a rabbit run in your garden, your drawing will be much smaller than the actual thing.

To convert the size of your plan to the actual size, you have to decide on a scale. e.g. 1 cm on your plan may represent 2 metres for your actual object. So if you had a plan drawing which measured 4cm long x 6cm wide, you would convert it like this:

4cm represents 4 x 2m = 8m
6cm represents 6 x 2m = 12m

If you know the size of the real object and you want to be able to make a drawing then you need to divide.

8m is represented by 8cm ÷ 2 = 4cm
12m is represented by 12cm ÷ 2 = 6cm

Scale – your turn

Using the rules on the previous page, draw these rectangular pictures of a hall and a classroom, then calculate the size of the real rooms.

1. The large hall is shown by a rectangle 6cm long and 8cm wide. Draw the rectangle. The drawing uses the scale 1cm represents 6m. Now work out the size of the actual room.
2. The classroom is shown as a rectangle 4cm long and 3.5cm wide. Draw the rectangle. The drawing uses the scale 1cm represents 0.5m. Now work out the actual size of the classroom.

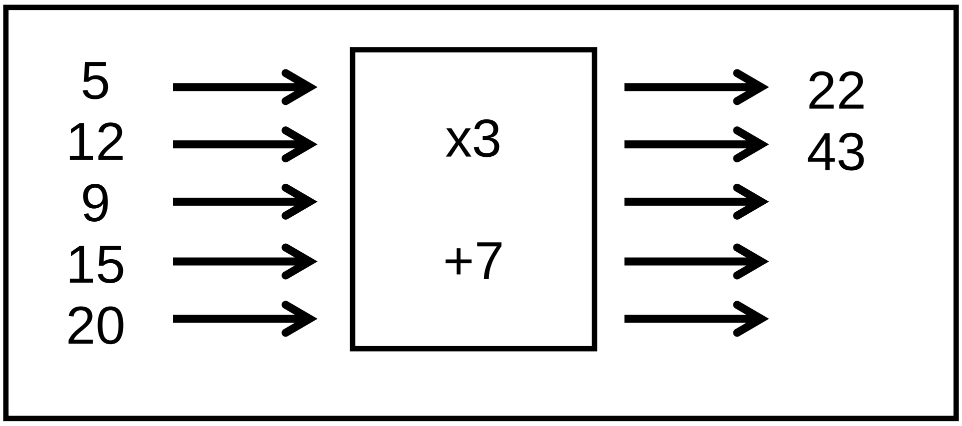
Now the Opposite

A farmer has 2 rectangular fields. He needs to draw them to scale.

1. The first field measures 45m long and 25m wide. Using a scale of 1cm represents 5m, make an accurate scale drawing of the field.
2. The second field measures 81m long and 36m wide. Using a scale of 1cm represents 9m, make an accurate scale drawing of the field.

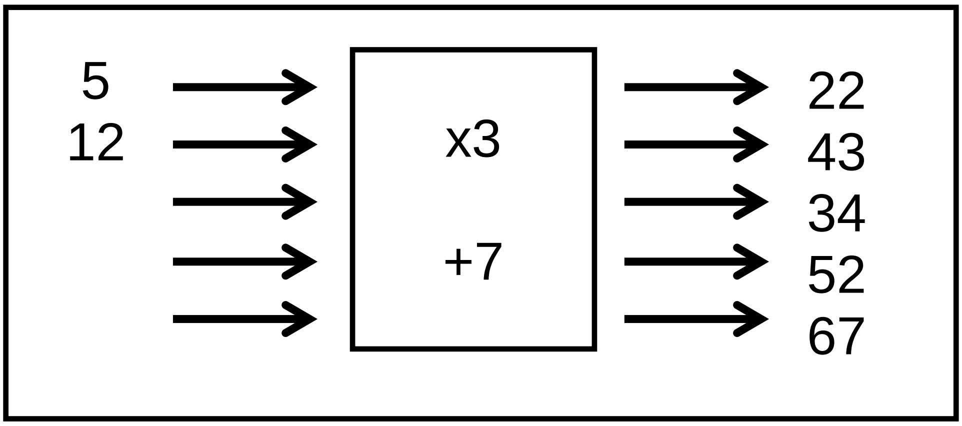
Function Machines

Function Machines are really just a shortcut way to write several sums.
e.g.



This really means 5 **x 3**= 15 **+ 7** = 22
12**x 3** = 36 **+ 7** =43

You can work out the rest of the answers in the same way.



Sometimes you need to work out answers to a function machine by starting at the end and working backwards. This time you need to do everything the **opposite** way round.
To find out what number went in to make the answer 34 you need to think backwards.

This really means 34 **– 7**= 27 ÷ **3** = 9

You can work out the rest of the answers in the same way.

Algebra

Sometimes you will see a number puzzle that looks very tricky.
e.g. $a = 6$ $b = 3$
$3a + b = c$ Find the value of c.

The work you may need to do is very similar to work that you did in Key Stage 1.
e.g. $4 \times 5 = \boxed{}$ or
$2 + \boxed{} = 10$ or
$\boxed{} \times 4 = 12$

The only real difference is that the box $\boxed{}$ has been replaced by a number.
e.g. $4 \times 5 = a$
In this example, you can easily see $a = 20$

Look back at the question at the top of the page.
$3a + b = c$
You know that $b = 3$ so you can rewrite the sum as: $3a + 3 = c$

3a really means (3 sets of a) or (3 multiplied by a.)
If a is 6 then 3a must be $3 \times 6 = 18$

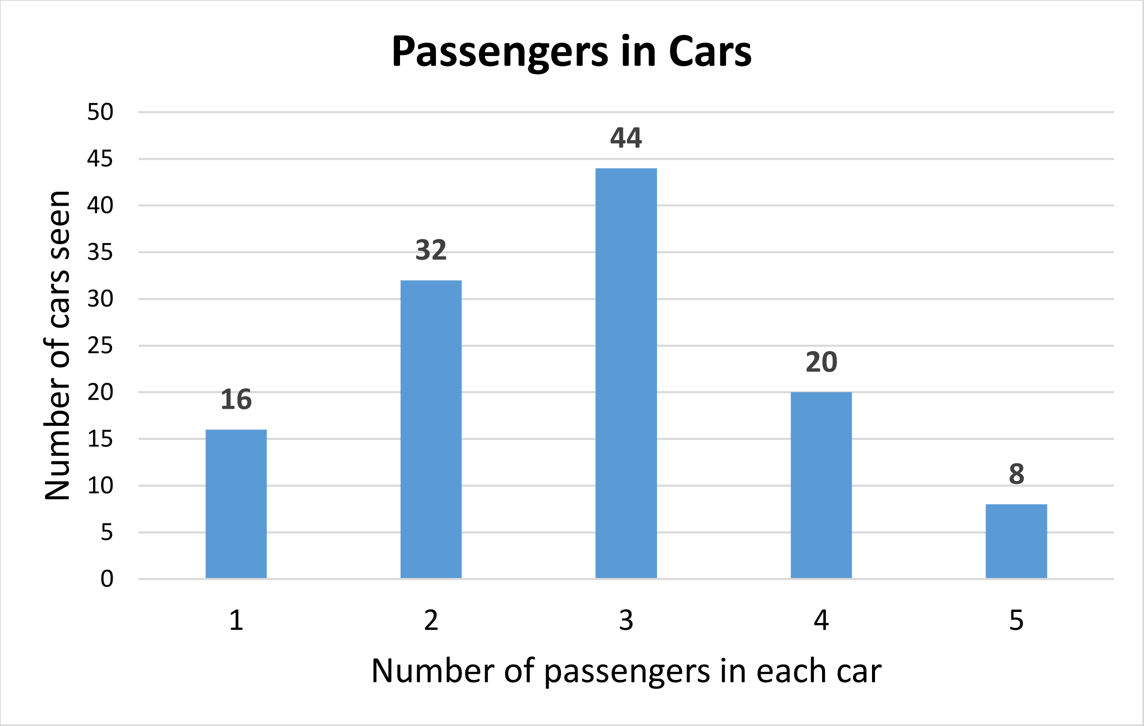
Now you can rewrite the sum as $18 + 3 = c$
$18 + 3 = 21$ so c must be 21.

Don't forget, in algebra the multiply sign is not used so if you see a number immediately before a letter then you need to multiply.
The division sign ÷ is also not used. $b \div 2$ would be shown as $^{b}/_{2}$.

Remember

2a means 2 times a
ab means a times b
$^{a}/_{2}$ means a divided by 2
$^{a}/_{b}$ means a divided by b

Data Handling



This bar chart shows the number of people who travelled to school by car in one day and how many people were in each car.

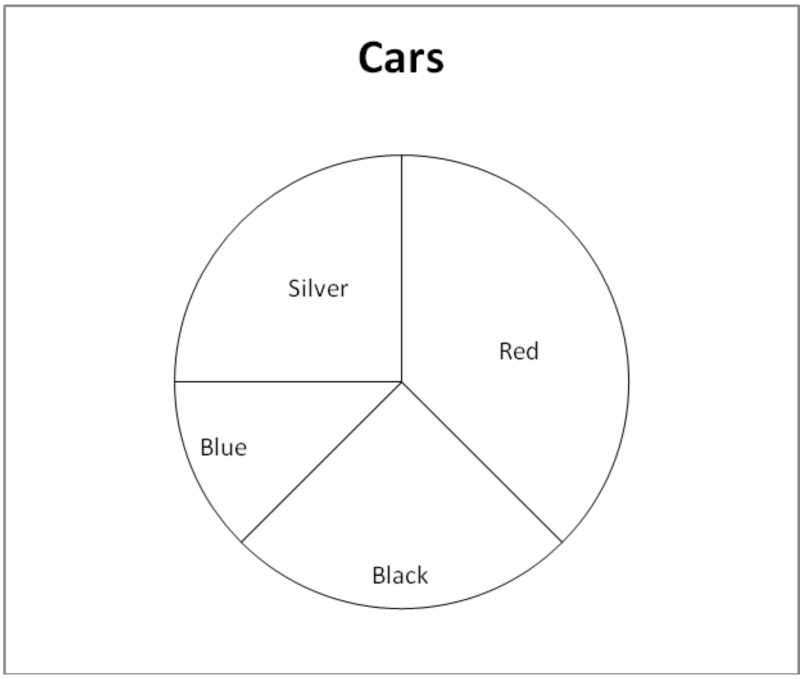
By reading the graph you can see:

16 cars had 1 person
32 cars had 2 people
44 cars had 3 people
20 cars had 4 people
8 cars had 5 people

If you are asked how a question such as – how many cars had 3 or more people in them, then you must remember to add up **all** the cars that had 3 or 4 or 5 people in them as they all had either 3 or more than 3 people.

Pie Charts

A pie chart is simply a different way to represent data. In this chart the survey is looking at the colour of the cars arriving at school. 120 cars were counted.



To calculate how many of each colour you first need to work out what fraction of the total is in each section. If you look closely at the chart you can see that it is divided into 8 sections which means that each single section contains 15 cars. (120 ÷8 = 15)
Rule lines across the pie chart if it helps you to work out the numbers more easily.

Now you can see that the blue section must contain $^1/_8$ of the total or 15 cars.

Silver and black both contain $^2/_8$ or ¼ of 120 which is 30 cars.

Red has $^3/_8$ of 120 which is 45 cars.

Long Multiplication

If you know your times tables very well then long multiplication will not be difficult. Look at the example below. 358 x 62 is really the same as writing two sums: 358 x 60   + 358 x 2

$$\begin{array}{ccc}
\text{H} & \text{T} & \text{U} \\
3 & 5 & 8 \\
\text{x}_1 & 6_1 & 2 \\
\hline
\end{array}$$

$$\begin{array}{ccccc}
 & & \text{H} & \text{T} & \text{U} \\
 & & 3 & 5 & 8 \\
 & & {}_3\text{x} & {}_4 6 & 0 \\
\hline
2 & 1 & 4 & 8 & 0
\end{array}
\qquad + \qquad
\begin{array}{ccc}
\text{H} & \text{T} & \text{U} \\
3 & 5 & 8 \\
\text{x}_1 & {}_1 & 2 \\
\hline
7 & 1 & 6
\end{array}$$

It is quicker to write both parts as one sum like this.

$$\begin{array}{cccccl}
 & & \text{H} & \text{T} & \text{U} & \\
 & & 3 & 5 & 8 & \\
 & & \text{x}_1 & 6_1 & 2 & \\
\cline{3-5}
 & & 7 & 1 & 6 & \text{(U)} \\
+2 & 1_1 & 4 & 8 & 0 & \text{(T)} \\
\cline{1-5}
2 & 2 & 1 & 9 & 6 &
\end{array}$$

Remember, when you’re multiplying by a number in the TENS column, you’re not going to have any number in the units column so don’t forget to start that part of the sum by writing a 0 in the units. The pattern continues when you multiply by hundreds, thousands etc.

Long Division

As with long multiplication, if you know your times tables very well then long division should not be too tricky. Long division works the same way as ordinary division but using bigger numbers. To make it easier the sum is split into different chunks.
You can do a long division style sum with smaller numbers too! Look at the example below.

$$\begin{array}{r|l} & \;\;2\;3 \\ \cline{2-2} 6 & 1\;3^{1}8 \end{array}$$

This simple sum can be done in the same way as long division.

$$\begin{array}{r|l} & \;\;2\;3 \\ \cline{2-2} 6 & 1\;3\;8 \\ & \underline{-1\;2}\downarrow \\ & \;\;\;1\;8 \\ & \underline{\;-1\;8} \\ & \;\;\;0\;0 \end{array}$$

First you think how many 6s in 13?

6 x 2 = 12

Write 12 below the 13 part and subtract which leaves 1.
Now bring down the 8 from the top to make 18 at the bottom.
Now think, how many 6s in 18?
You know 6 x 3 = 18 so you can complete your answer.

Dividing by bigger numbers works exactly the same way.

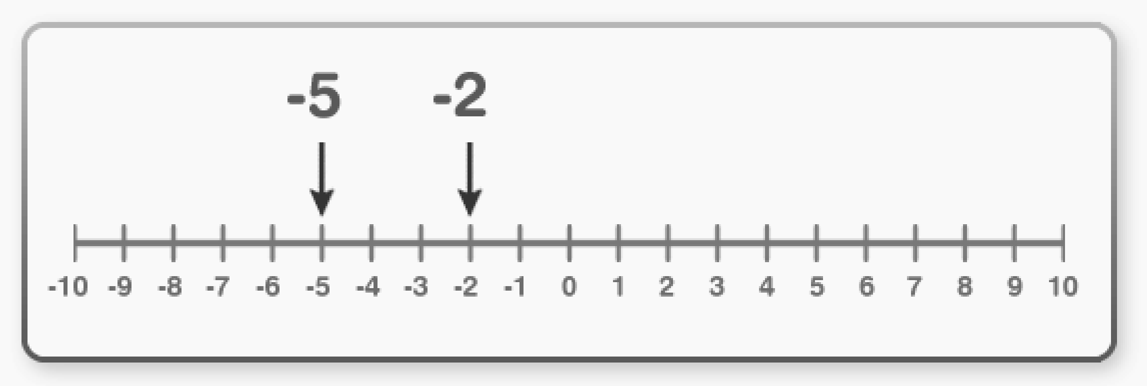
$$\begin{array}{r|l} & \;\;\;\;\;2\;5\;3 \\ \cline{2-2} 12 & 3\;\;0\;\;3\;\;6 \\ & \underline{-2\;4}\downarrow\;\;\big\downarrow \\ & \;\;\;\;\;6\;3 \\ & \underline{\;\;-6\;0} \\ & \;\;\;\;\;\;\;\;3\;\;6 \\ & \underline{\;\;\;\;\;-3\;\;6} \\ & \;\;\;\;\;\;\;\;0\;\;0 \end{array}$$

Negative Numbers

Negative numbers are numbers that are less than zero. e.g. – 5 really means 0 – 5 and 5 really means 0 + 5. These numbers might look the same but they are on opposite sides of zero.Negative numbers can be a little confusing.

It is important to remember that positive numbers get higher the more they move away from zero but negative numbers get smaller the more they move away from zero.

A number line is really useful to understand negative numbers.



As you can see from the picture above, -5 is a smaller number than -2 because it is three steps further away from zero. -2 – 3 = -5

If you started at -8 and added 4, you would reach -4.

If you started at -6 and added 6 you would reach 0.

If you start at 1 and subtract 8 then you would end up at -7.

Now it’s your turn. Write these numbers in order from smallest to largest.

-4 7 2 -12 16 14 -14 8 0 -20